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Virtual Aerodynamical Coefficients of a Rotating Satellite

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Nomenclature

a = semimajor axis
$\boldsymbol{a}$ = vector of orbital elements (a, e, i, M_0, ω, Ω)
C_D = drag coefficient
C_L = lift coefficient
E = eccentric anomaly
e = eccentricity
H = density scale height
i = inclination
M = mean anomaly
$\hat{P}$ = perifocal unit vector (radial)
$\hat{Q}$ = perifocal unit vector (transverse)
α = angle of attack
θ = true anomaly
ρ = atmospheric density
Ω = longitude of ascending node
ω = argument of perigee
ω_b = angular rate of satellite rotation

Introduction

A LOW satellite orbit is generally affected by the atmospheric drag. When the satellite has large panels, the drag depends strongly on the satellite's orientation. Moreover, a normal force (lift) may also act on the satellite. The coefficients of these forces are time-varying in general, since the angle of attack (the angle between a reference body line and the velocity vector) changes with time. This angle is constant in the particular case when the satellite's rotational spin and the orbital rate are identical, in circular orbit. This is a rare exception. Thus, the aerodynamical perturbations should be generally considered as time dependent. However, the satellite attitude history cannot in general be deduced from ground tracking data. It must be known to integrate the equation of motion with the proper aerodynamical coefficients. One approach to deal with this uncertainty is to perform a tedious numerical integration using various aerodynamical coefficients. Another approach is to consider the averaged orbital elements. In this case a representative value for the aerodynamical coefficients should be adopted. This issue is addressed in the following discussion.

The problem of satellite orbits in the atmosphere is treated extensively in the literature. The usual approach[1] is to consider the drag coefficient as constant, substitute the drag into the Gauss variational equations, and perform averaging. The time, or the orbital anomaly (θ, E, or M), appears in the integrands because of the ellipticity as well as the atmospheric modeling. King-Hele[2] presented a comprehensive resource for the influence of the atmospheric characteristics on the averaged orbital elements. Liu[3] focused his review on various perturbation methods for constant drag coefficients and simple atmospheric models. Because of large uncertainties of the model, he warns from falling into the trap of including higher-order effects. Roth[4] and Ashenberg and Broucke[5] included the lifting forces. Reference 4 assumed constant coefficients, but Ref. 5 challenged this assumption for the case of rotating fragments. Roy[6] suggested taking the average cross-sectional area when the satellite attitude is changing. However, this is a problematic matter, as was first pointed out by Cook[7] for the case of sign-changing lift. Liu made a comment doubting the validity of the constant-drag-coefficient assumption for long-term prediction. The current research confronts this issue and tries to present a convenient way of modeling the varying aerodynamical coefficients in the average sense. To focus on the effects of the time-varying aerodynamics, the other perturbations will be eliminated. Also, the simplest, exponential-density atmospheric model is adopted:

$$\rho = \rho_{p_0} \exp[-\gamma(1 - \cos E)] \tag{1}$$

where ρ_{p_0} is the initial density at perigee and $\gamma \overset{\text{def}}{=} ae/H$. Typical satellite shapes for the current presentation are dominated by flat plates, for example, satellites with solar panels. Spherelike shapes are characterized by almost constant aerodynamical coefficients, and thus may be considered as a degenerate group. The aerodynamical coefficients are evaluated by applying the free-molecular-flow theory.[8] For the sake of a qualitative argument, let C be an aerodynamical coefficient, consisting of constant and time-dependent parts: $C = C_0 + C_1(t)$. The average influence on the orbital elements may be written as $\Delta\boldsymbol{a} = \Delta\boldsymbol{a}_c + \eta\Delta\boldsymbol{a}_r$, where the subscripts c and r refer to constant and rotating coefficients, η depends on the satellite shape, and $\eta \in [0, 1]$. Actually η is the sensitivity of the orbit to aerodynamical changes. The aerodynamical forces (tangential, normal, orthogonal) are $F_{\{T,N,W\}} = \{-B_D, -B_L, B_W\}v^2$, where $B_{D,L,W} = \frac{1}{2}(A/m)\rho F C_{\{D,L,W\}}$, where F contains the atmospheric rotation. Note that the aerodynamical coefficients are time-varying functions although they are defined as functions of the angle of attack. The reason is that the angle of attack is a time-varying function from two kinematical causes: the satellite attitude kinematics and the directional variation of the relative velocity vector. The solution approach is the following: expressing the angle of attack in terms of the orbital anomaly, substituting the aerodynamical forces in the Gauss variational equations, changing the independent variable from true anomaly to eccentric anomaly ($\theta \to E$), and finally taking the average along an orbit.

Averaged Orbital Elements

Let $\dot{\boldsymbol{a}} = f(\boldsymbol{a}, t)$ be the variational equations. The averaged equations are

$$\dot{\bar{\boldsymbol{a}}} = \frac{1}{2\pi}\int_0^{2\pi} f(\boldsymbol{a}, t)\,\mathrm{d}M \tag{2}$$

and the average increment of the orbital element per orbit is

$$\Delta\boldsymbol{a} = \frac{r}{na}\int_0^{2\pi} f(\boldsymbol{a}, E)\,\mathrm{d}E \tag{3}$$

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Denoting the averaging operator as $\langle F\rangle = \int_0^{2\pi} F\,dE$, the average increments are the following:

$$\Delta a = -2a^2\langle K_D^a B_D\rangle$$
$$\Delta e = -2p\left(\langle K_D^e B_D\rangle - \frac{1}{2\sqrt{1-e^2}}\langle K_L^e B_L\rangle\right)$$
$$\Delta i = a\left(\frac{\cos\omega}{\sqrt{1-e^2}}\langle K_{1W}^i B_W\rangle - \sin\omega\langle K_{2W}^i B_W\rangle\right)$$
$$\Delta\omega = -\frac{a}{e}\left(2\sqrt{1-e^2}\langle K_D^\omega B_D\rangle + \langle K_L^\omega B_L\rangle\right) - a\frac{\cos i}{\sin i}\left(\frac{\sin\omega}{\sqrt{1-e^2}}\langle K_{1L}^\omega B_W\rangle + \cos\omega\langle K_{2W}^\omega B_W\rangle\right)$$
$$\Delta\Omega = \frac{a}{\sin i}\left(\frac{\sin\omega}{\sqrt{1-e^2}}\langle K_{1W}^\Omega B_W\rangle + \cos\omega\langle K_{2W}^\Omega B_W\rangle\right) \tag{4}$$

where

$$K_D^a = \sqrt{\frac{(1+e\cos E)^3}{1-e\cos E}}$$
$$K_D^e = \frac{\cos E\sqrt{1+e\cos E}}{\sqrt{1-e\cos E}}, \qquad K_L^e = \sin E\sqrt{1-e^2\cos E}$$
$$K_D^\omega = \sqrt{\frac{1+e\cos E}{1-e\cos E}}\sin E$$
$$K_L^\omega = \sqrt{1-e^2\cos^2 E}\,(\cos E + e)$$
$$K_{1W}^i = K_{1L}^\omega = K_{1W}^\Omega = (1-e^2)\cos E - e\sin^2 E$$
$$K_{2W}^i = K_{2W}^\omega = K_{2W}^\Omega = (1+e\cos E)\sin E \tag{5}$$

The aerodynamical perturbations contribute secular variations when $\langle K_j^a(E)B_j(E)\rangle \neq 0$. The K_j are given by the orbit, and the B_j are controlled by the satellite attitude history. Time-varying aerodynamical coefficients will obviously lead to different results from the classical results regarding constant coefficients. Moreover, there is no hope for expecting a general conclusion. Each aerodynamical variation must be considered separately.

Virtual Aerodynamical Coefficients

The secular variation of the orbital elements is not unique, but depends on the particular rotational kinematics. Each case requires a different solution. Suppose that one wants to avoid the tedious procedure of averaging by applying the classical solution for constant coefficients. A typical scenario is orbital debris. Given a large ensemble of rotating fragments, the envelope of the atmospheric spread is determined by the particular fragments with rotation rate that maximize the secular variation. Thus, if one knows the virtual constant coefficients that give rise to the same variation as the original coefficients, then a tedious computation may be avoided. For the purpose of defining the virtual coefficients, let Δa_j be the secular change of the orbital element a_j due to the aerodynamical coefficient $C_i(E)$. The secular change can be formulated as $\Delta a_j = G\int_0^{2\pi} K_i^j C_i \exp(\gamma\cos E)\,dE$, where G is a constant depending on A, m, ρ_0, γ, and some osculating elements. On the other hand, for a constant coefficient C_i^c, $\Delta a_j^c = GC_i^c F^c(e,\gamma)$, where F^c is a precomputed function, $F^c = \int_0^{2\pi} K_i^j(E)\exp(\gamma\cos E)\,dE$. Let $\bar{C}_i$ be the virtual aerodynamical coefficient, that is, $\Delta a_j = G\bar{c}_i F^c$. Equating Δa_j with Δa_j^c yields

$$\bar{C}_i = \frac{\int_0^{2\pi} K_i^j(E)C_i(E)e^{\gamma\cos E}\,dE}{F^c(e,\gamma)} \tag{6}$$

Note that the aerodynamical coefficients are basically functions of the angle of attack, which is time dependent. It is convenient to expand the coefficients as Fourier series. The structure of the expansion is subjected to the following characteristics: C_D is an even function of the angle of attack α defined in the interval $\alpha = [0,\pi]$, and C_L is an odd function of α defined in the same interval. Therefore, an expansion in α can be formulated as

$$C_D(\alpha) = C_{D0} + \sum_{n=0}^{\infty} a_{2n}\cos 2n\alpha, \qquad C_L(\alpha) = \sum_{n=1}^{\infty} b_{2n}\sin 2n\alpha \tag{7}$$

The above structure is very convenient for further applications because of the trigonometric functions that play a natural role in the averaging.

The next unavoidable question is how sensitive the virtual aerodynamical coefficients are to the attitude kinematics. The answer depends on three factors. First, the satellite shape: η must be big enough. Second, the satellite kinematics, consisting of the satellite angular rate (spin) and the time history of angle of attack. The methodical example in the following section will demonstrate this issue for an aerodynamically idealized satellite.

Application

Taking the extreme case, a flat plate ($\eta = 1$), the aerodynamical coefficients for rarefied flow[8] are expanded in Fourier series. In addition, a satellite with spin $\frac{1}{2}$ in a nearly circular orbit is considered. In other words, the satellite rotates half a revolution per orbit. This particular case causes the highest changes in orbital elements. The following three-term expansion gives a good approximation:

$$\begin{aligned} C_D(\alpha) &\approx A_0 + A_1\cos 2\alpha + A_2\cos 4\alpha + A_3\cos 6\alpha \\ C_L(\alpha) &\approx B_1\sin 2\alpha + B_2\sin 4\alpha + B_3\sin 6\alpha \end{aligned} \tag{8}$$

The corresponding Fourier coefficients are

$$A_0 = \left(\frac{4}{\pi} + \frac{v_r}{v_i}\right)\sigma + \frac{16}{3\pi}(1-\sigma)$$
$$A_1 = -\left(\frac{8}{3\pi} + \frac{v_r}{v_i}\right)\sigma - \frac{32}{5\pi}(1-\sigma) \tag{9a}$$
$$A_2 = -\frac{8}{15\pi}\sigma + \frac{32}{35\pi}(1-\sigma), \qquad A_3 = -\frac{8}{35\pi}\sigma$$

and

$$B_1 = \frac{v_r}{v_i}\sigma + \frac{64}{15\pi}(1-\sigma), \qquad B_2 = -\frac{128}{105\pi}(1-\sigma)$$
$$B_3 = -\frac{64}{315\pi}(1-\sigma) \tag{9b}$$

where v_r and v_i are the incident speed and the reemission speed of the molecules, respectively, and σ is the accommodation coefficient; typically[9] $\sigma = 0.85$. To express the aerodynamical coefficients in terms of the eccentric anomaly, the following steps are taken. First, the angle of attack is evaluated from $\cos\alpha = \hat{\boldsymbol{b}}\cdot\hat{\boldsymbol{V}}$ and expanded in the small eccentricity. Then, Kepler's equation is substituted in to obtain an expression in terms of the eccentric anomaly: $\alpha = \frac{1}{2}(E-\psi) + \frac{1}{2}e\sin E + \frac{1}{4}e^2\sin 2E + \cdots$. Here ψ indicates the angle-of-attack phase, defined as follows. Let $\hat{\boldsymbol{b}}$ denote a reference direction of the satellite such that α is the angle between $\hat{\boldsymbol{b}}$ and the velocity. Then ψ is defined from $\hat{\boldsymbol{b}} = -\sin(\omega_b t - \psi - \omega)\hat{\boldsymbol{P}} + \cos(\omega_b t - \psi - \omega)\hat{\boldsymbol{Q}}$. Substituting into the aerodynamical coefficients and expanding the trigonometrical terms yields

$$\begin{aligned} C_D(E,\psi) &= \left(A_0 - \tfrac{1}{2}eA_1\cos\psi\right) \\ &+ (A_1\cos\psi - eA_2\cos 2\psi)\cos E \\ &+ \tfrac{1}{2}(eA_1\cos\psi + 2A_2\cos 2\psi - 3eA - 3\cos 3\psi)\cos 2E \\ &+ (eA_2\cos 2\psi + A_3\cos 3\psi)\cos 3E + \tfrac{3}{2}eA_3\cos 3\psi\cos 4E \\ &+ (A_1\sin\psi + eA_2\sin 2\psi)\sin E \\ &+ \tfrac{1}{2}(eA_1\sin\psi + 2A_2\sin 2\psi - 3eA_3\sin 3\psi)\sin 2E \\ &+ (eA_2\sin 2\psi + A_3\sin 3\psi)\sin 3E \\ &+ \tfrac{3}{2}eA_3\sin 3\psi\sin 4E \end{aligned} \tag{10a}$$

$$
\begin{aligned}
C_L(E,\psi) &= \tfrac{1}{2}eB_1\sin\psi + (-B_1\sin\psi + B_2\sin 2\psi)\cos E \\
&\quad - \left(\tfrac{1}{2}eB_1\sin\psi + B_2\sin 2\psi - \tfrac{3}{2}eB_3\sin 3\psi\right)\cos 2E \\
&\quad - (eB_2\sin 2\psi + B_3\sin 3\psi)\cos 3E - \tfrac{3}{2}eB_3\sin 3\psi\cos 4E \\
&\quad + (B_1\cos\psi - eB_2\cos 2\psi)\sin E \\
&\quad + \tfrac{1}{2}(eB_1\cos\psi + 2B_2\cos 2\psi - 3eB_3\cos 3\psi)\sin 2E \\
&\quad + (eB_2\cos 2\psi + B_3\cos 3\psi)\sin 3E \\
&\quad + \tfrac{3}{2}eB_3\cos 3\psi\sin 4E
\end{aligned} \tag{10b}
$$

Note the mixed harmonics in the above expansion and the strong dependence on the satellite attitude ψ. Also, the lift coefficient is not purely periodic any more, but has a bias because of the eccentricity.

For further applications, let us express the aerodynamical coefficients as

$$
\begin{aligned}
C_D(E,\psi) &= \tilde{A}_0(\psi) + \sum_{k=1}^{4}\tilde{A}_k(\psi)\cos kE + \tilde{B}_k(\psi)\sin kE \\
C_L(E,\psi) &= \tilde{C}_0(\psi) + \sum_{k=1}^{4}\tilde{C}_k(\psi)\cos kE + \tilde{D}_k(\psi)\sin kE
\end{aligned} \tag{11}
$$

with coefficients corresponding to the above expansion. The evaluation of the virtual coefficients is not always unique, but depends on particular considerations. For example, $\bar{C}_D$ is naturally evaluated from the semimajor-axis decay, and $\bar{C}_L$ and $\bar{C}_W$ may be evaluated from the perigee motion and the inclination variation, respectively. Therefore

$$
\begin{aligned}
\bar{C}_D(\psi) &= \frac{\int_0^{2\pi} K^a_{D(E)} C_{D(E,\psi)} e^{\gamma\cos E}\,\mathrm{d}E}{\int_0^{2\pi} K^a_{D(E)} e^{\gamma\cos E}\,\mathrm{d}E} \\
&= \frac{\tilde{A}_0 I_0 + \tilde{A}_1 I_1 + e(\tilde{A}_0 I_1 + \tilde{A}_1 I_0 + \tilde{A}_2 I_1)}{I_0 + 2eI_1} + O(e^2)
\end{aligned} \tag{12a}
$$

$$
\begin{aligned}
\bar{C}_L(\psi) &= \frac{\int_0^{2\pi} K^\omega_{L(E)} C_{L(E,\psi)} e^{\gamma\cos E}\,\mathrm{d}E}{\int_0^{2\pi} K^\omega_{L(E)} e^{\gamma\cos E}\,\mathrm{d}E} \\
&= \frac{\tilde{C}_0 I_1 + \frac{1}{2}\tilde{C}_1 I_0 + \frac{1}{2}\tilde{C}_2 I_1 + e(\tilde{C}_0 I_0 + \tilde{C}_1 I_1)}{I_1 + eI_0} + O(e^2)
\end{aligned} \tag{12b}
$$

and

$$
\begin{aligned}
\bar{C}_W(\psi) &= \frac{\int_0^{2\pi} K_{1(E)} C_{W(E,\psi)} e^{\gamma\cos E}\,\mathrm{d}E}{\int_0^{2\pi} K_{1(E)} e^{\gamma\cos E}\,\mathrm{d}E} \\
&= \frac{\left(\tilde{C}_0 + \frac{1}{2}\tilde{C}_2\right) I_1 + \frac{1}{2}\tilde{C}_1 I_0 - \frac{1}{2}e\left(\tilde{C}_0 I_0 + \frac{1}{2}\tilde{C}_1 I_1\right)}{I_1 - \frac{1}{2}eI_0} + O(e^2)
\end{aligned} \tag{12c}
$$

where $\tilde{A}_k$, $\tilde{C}_k$ are functions of ψ, and I_n is the Bessel function, of the first kind of order n:

$$
I_n(\gamma) = \frac{1}{2\pi}\int_0^{2\pi}\exp(\gamma\cos E)\cos nE\,\mathrm{d}E \tag{13}
$$

Figure 1 shows the virtual coefficients as functions of the attitude phase angle at epoch ψ, for $e = 0.01$. The virtual drag coefficient is higher near $\psi = \pi$. That is expected, because this corresponds to the situation where the plate is nearly perpendicular to the flow at perigee, where the dissipation is the highest. The variation of $\bar{C}_D$ depends on the eccentricity as follows:

$$
\langle C_D\rangle_\psi = \langle C_D\rangle_\alpha + ef(\psi) \tag{14}
$$

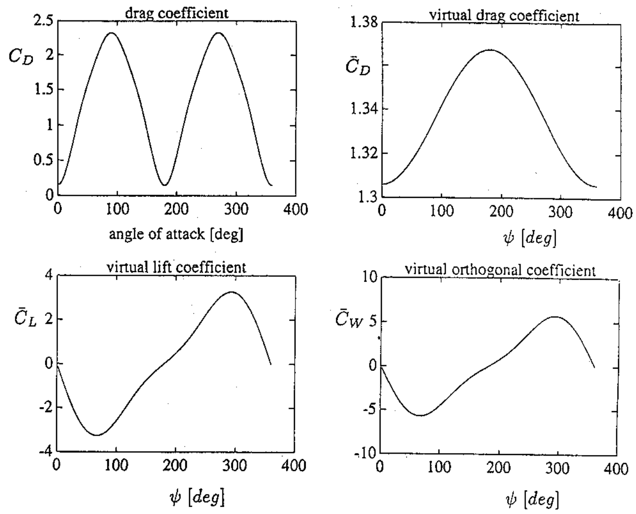

Fig. 1 Virtual aerodynamical coefficients.

i.e., the virtual drag coefficient correspond to the average drag coefficient with fluctuation of order e. The virtual side (lift and orthogonal force) coefficients are amplified because of the spin $\frac{1}{2}$. The symmetry of these coefficients with respect to the major axis of the orbit (near $\psi = 90$ and 270 deg) maximizes the value of the virtual coefficients, whereas the asymmetry near $\psi = 0$ and 180 deg cancels the virtual side coefficients.

Concluding Remarks

This Note suggests a method for dealing with the atmospheric influence of an aerodynamically rotating satellite on its orbit. Although the propagation of the orbital elements may be evaluated numerically, for long-term prediction, averaging is the proper way. The trouble is that the orbital propagation strongly depends on the aerodynamical rotation. Defining the virtual aerodynamical coefficients gives an analogy to the classical averaging for constant coefficients and indicates the actual influence of each coefficient. It is essential to be aware of the difference between the virtual and the average coefficients. Thus, one should generally avoid applying average aerodynamical coefficients for orbital prediction. The exception is for very high-spin satellites. The concept of virtual aerodynamical coefficients is mostly applicable for estimating orbital uncertainties due to aerodynamical variations. Aerodynamical spread because of an ensemble of spinning fragments is a typical example. Further work, to be published soon, will deal with a combination of varying aerodynamics with the periodic solar day–night effects.

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